Derivatives of Trigonometric Functions

Some facts about trigonometric functions:

(a) $\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$

(b) $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

(c) $\lim_{h\to 0}\left(\frac{\cos(h)-1}{h}\right) = 0$

(d) $\lim_{h\to 0}\left(\frac{\sin(h)}{h}\right) = 1$

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1. Goal: to find the derivative of $y = \sin(x)$.

$$\frac{d}{dx}\left[\sin(x)\right] = \lim_{h\to 0}\frac{\boxed{}}{\boxed{}}$$ (Definition of Derivative)

$$= \lim_{h\to 0}\frac{\boxed{}}{\boxed{}}$$ (Trig Fact (a) above)

$$= \lim_{h\to 0}\left(\sin(x)\cdot\frac{\boxed{}}{\boxed{}} + \cos(x)\cdot\frac{\boxed{}}{\boxed{}}\right)$$ (Split in Two)

$$= \sin(x)\cdot \underline{} + \cos(x)\cdot \underline{}$$ (Trig Facts (c) & (d) Above)

$$= \underline{}$$

Conclusion: $\frac{d}{dx}\left[\sin(x)\right] = \underline{}$

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2. Goal: Using similar methods, show that:
$\frac{d}{dx}\left[\cos(x)\right] =$

$\boxed{= -\sin(x)}$

3. Goal: Find the derivative of $\tan(x)$

$\frac{d}{dx}\left[\tan(x)\right]$	$=$	$\frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right]$	(Why? ________________________)
	$=$	$\frac{\boxed{}}{\boxed{}}$	(The Quotient Rule)
	$=$	$\frac{\boxed{}}{\boxed{}}$	($\sin^2(x) + \cos^2(x) = 1$)
	$=$	$\sec^2(x)$	(Why? ________________________)

Conclusion: $\frac{d}{dx}\left[\tan(x)\right] =$ __________________

4. Similarly, we can show:

$$\frac{d}{dx}\left[\cot(x)\right] = -\csc^2(x)$$

$$\frac{d}{dx}\left[\sec(x)\right] = \sec(x)\tan(x)$$

$$\frac{d}{dx}\left[\csc(x)\right] = -\csc(x)\cot(x)$$

5. Find $\frac{dy}{dx}$

   (a) $y = \frac{\cos(x)}{x^2-3x+2}$

   (b) $y = e^x \tan(x)$

   (c) $y = \sin^2(x)$

   (d) $y = \frac{3x^2-5x}{\cot(x)}$

   (e) $y = \sec(x) \cdot \cos(x)$ [2 methods]

   (f) $y = \frac{\tan(x)}{\cot(x)}$ [2 methods]

6. Homework: ____________________________