Derivatives of Inverse Functions

Some facts about inverse functions:

(a) $y = \log_b(x) \iff b^y = x$

(b) $y = \sin^{-1}(x) \iff \sin(y) = x$

(c) $\cos\theta = \sqrt{1 - \sin^2\theta}$ if $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

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1. Goal: to find the derivative of $y = \ln(x)$.

i.	$y = \ln(x)$	$\Longrightarrow \quad e^{\square} = \_\_\_\_\_$	(Fact (a) above)
ii.		$\Longrightarrow \quad e^{\square} = \_\_\_\_\_$	(Substitute $y = f(x)$)
iii.		$\Longrightarrow \quad e^{\square} \cdot \_\_\_\_\_ = \_\_\_\_\_$	(Implicit differentiation)
iv.		$\Longrightarrow \quad \_\_\_\_\_ \cdot \frac{dy}{dx} = \_\_\_\_\_$	(Substitute using **i.** and $\frac{dy}{dx} = f'(x)$)
v.		$\Longrightarrow \quad \frac{dy}{dx} = \_\_\_\_\_$	(Divide)

Conclusion: $\frac{d}{dx}\big[\ln(x)\big] = \_\_\_\_\_$

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2. Goal: Using similar methods, find $\frac{d}{dx}\big[\sin^{-1}(x)\big]$. (Recall that if $y = \sin^{-1}(x)$, then $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.)

i.	$y = \sin^{-1}(x)$	$\Longrightarrow \quad \sin(\_\_\_\_\_) = \_\_\_\_\_$	(Fact (b) above)
ii.		$\Longrightarrow \quad \sin(\_\_\_\_\_) = \_\_\_\_\_$	(Substitute $y = f(x)$)
iii.		$\Longrightarrow \quad \cos(\_\_\_\_\_) \cdot \_\_\_\_\_ = \_\_\_\_\_$	(Implicit differentiation)
iv.		$\Longrightarrow \quad \cos(\_\_\_\_\_) \cdot \frac{dy}{dx} = \_\_\_\_\_$	(Substitute $y = f(x)$ & $\frac{dy}{dx} = f'(x)$)
v.		$\Longrightarrow \quad \sqrt{\square} \cdot \frac{dy}{dx} = \_\_\_\_\_$	(Fact (c) above)
vi.		$\Longrightarrow \quad \sqrt{\square} \cdot \frac{dy}{dx} = \_\_\_\_\_$	(Substitute using **i.**)
vii.		$\Longrightarrow \quad \frac{dy}{dx} = \frac{\square}{\square}$	(Divide)

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3. Homework: _____