Integration by Parts

Recall the **Product Rule:**

$$\frac{d}{dx}\Big[f(x)\cdot g(x)\Big] = \_\_\_\_\_\_\_\_ \cdot \_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_ \cdot \_\_\_\_\_\_\_\_$$

This implies:

$$\int \_\_\_\_\_\_\_\_ \cdot \_\_\_\_\_\_\_\_ dx + \int \_\_\_\_\_\_\_\_ \cdot \_\_\_\_\_\_\_\_ dx = \_\_\_\_\_\_\_\_ \cdot \_\_\_\_\_\_\_\_ + C$$

or:

$$\int f(x)\cdot g'(x)\ dx = \_\_\_\_\_\_\_\_ \cdot \_\_\_\_\_\_\_\_ - \int \_\_\_\_\_\_\_\_ \cdot \_\_\_\_\_\_\_\_ dx$$

Substituting: $\left\{ \begin{array}{rclcrcl} u & = & f(x) & \quad & dv & = & g'(x)\ dx \\ du & = & f'(x)\ dx & & v & = & g(x) \end{array} \right\}$, we get:

$$\int \_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_ - \int \_\_\_\_\_\_\_\_ .$$

This is called *Integration By Parts* (IBP).

Example 1: $\int x\cos x\ dx = \int u\cdot dv$, $\quad \left(\begin{array}{lrclrcl} \textit{Substitute}: & u & = & x & dv & = & \cos x\ dx \\ & du & = & \_\_\_\_\_\_\_\_ & v & = & \_\_\_\_\_\_\_\_ \end{array} \right)$

$$= \_\_\_\_\_\_\_\_ - \int \_\_\_\_\_\_\_\_$$

$$=$$

Example 2: $\int x^3 \ln x\ dx = \int u\cdot dv$, $\quad \left(\begin{array}{lrclrcl} \textit{Substitute}: & u & = & \ln x & dv & = & x^3\ dx \\ & du & = & \_\_\_\_\_\_\_\_ & v & = & \_\_\_\_\_\_\_\_ \end{array} \right)$

$$= \_\_\_\_\_\_\_\_ - \int \_\_\_\_\_\_\_\_$$

$$=$$

Example 3: $\int \arctan x \; dx = \int u \cdot dv,$ $\left(\begin{array}{lccccc} \textit{Substitute}: & u & = & \arctan x & dv & = & dx \\ & du & = & \rule{3cm}{0.4pt} & v & = & \rule{3cm}{0.4pt} \end{array} \right)$

$$= \rule{3cm}{0.4pt} - \int \rule{3cm}{0.4pt}$$

$$=$$

(You will need to use a u–Substitution to finish this problem.)

Example 4: $\int x^2 e^x \; dx = \int u \cdot dv,$ $\left(\begin{array}{lccccc} \textit{Substitute}: & u & = & \rule{3cm}{0.4pt} & dv & = & \rule{3cm}{0.4pt} \\ & du & = & \rule{3cm}{0.4pt} & v & = & \rule{3cm}{0.4pt} \end{array} \right)$

$$= \rule{3cm}{0.4pt} - \int \rule{3cm}{0.4pt}$$

$$=$$

(You will need to use IBP twice to finish this problem.)