Integration by Trigonometric Substitution

We now study a substitution "trick" employed to eliminate square roots in an integral. The trick is based on the Pythagorean Identity $\sin^2\theta + \cos^2\theta = 1$ or one of its equivalent identities discussed last section. The trick is summed up in the following table:

Expression	**Substitution**	**Identity**
$\sqrt{a^2 - x^2}$	$x = a \cdot \sin\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a \cdot \tan\theta$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a \cdot \sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

After integrating, a reverse substitution will be needed to return to the variable x.

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1. $\int \sqrt{a^2 - x^2}\ dx$

2. Find the area of a circle of radius r. *(finally!)*

3. $\int \frac{\sqrt{x^2-9}}{x}\ dx$

4. $\int \frac{x}{\sqrt{3-x^2}}\ dx$

5. $\int \frac{dx}{x^2\sqrt{x^2+36}}$

6. $\int_0^{2/3} \frac{x^3}{(9x^2+4)^{3/2}}\ dx$

7. $\int \sqrt{16+6x-x^2}\ dx$

HW __________