Integration by the Method of Partial Fractions

Consider the difference of two rational functions:

$$\frac{3}{x-2} - \frac{2}{2x+1} = \frac{3}{x-2} \cdot \frac{(\qquad\qquad)}{(\qquad\qquad)} - \frac{2}{2x+1} \cdot \frac{(\qquad\qquad)}{(\qquad\qquad)}$$

$$= \frac{(\qquad\qquad)}{(x-2)(2x+1)} - \frac{(\qquad\qquad)}{(x-2)(2x+1)}$$

$$= \frac{(\qquad\qquad\qquad\qquad)}{(x-2)(2x+1)}$$

$$= \frac{(\qquad\qquad\qquad\qquad)}{(\qquad\qquad\qquad\qquad)}$$

Therefore,

$$\int \frac{4x+7}{2x^2-3x-2}dx = \int \frac{(\qquad\qquad)}{(\qquad\qquad)}dx - \int \frac{(\qquad\qquad)}{(\qquad\qquad)}dx$$

$$=$$

This is called the *Method of Partial Fractions.* Some variant of this method can be applied to any function $f(x) = \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials and $deg(P) < deg(Q)$.

Example 1: What if $deg(P) \geq deg(Q)$?

$$\int \frac{x^3-3x-5}{x-2}dx =$$

It is clear, then, that we can really apply the above kind of idea to any rational function, so long as we can figure out how to decompose the rational function into its component partial fractions. Lets start with one we have already seen.

Example 2:

$$\int \frac{4x+7}{2x^2-3x-2}dx =$$

Example 3:

$$\int \frac{1}{x^3 - x} dx =$$

When a factor appears in the denominator with a power greater than one, a slightly different approach is needed.

Example 4:

$$\int \frac{1}{x^3 - x^2} dx =$$

Example 5:

$$\int \frac{2x^4 + x^3 - 3x - 1}{x^3 + 2x^2 + x} dx =$$