

Taylor & MacLaurin Series
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Let's imagine that e^x can be written as a polynomial (actually, it cannot, but that doesn't stop us from imagining that it can). Then

$$e^x = c_0 + c_1x + c_2x^2 + c_3x^3 + \dots + c_nx^n$$

for some appropriate integer $n \geq 0$. For the moment, let's pretend that $n = 0$, so $e^x = c_0$. Focusing on what is happening to e^x at $x = 0$, we get $c_0 = \underline{\hspace{2cm}}$. Try graphing both (that is, e^x and the value you got for c_0) on your graphing calculator, with $xMin = -3$, $xMax = 3$, $yMin = -1$, $yMax = 7$. Describe what you find.

They are clearly not the same functions, so let's move next to $n = 1$. Then since $e^x = c_0 + c_1x$, we expect that their derivatives are also equal. Differentiate both sides and plug in $x = 0$ to find $c_1 = \underline{\hspace{2cm}}$. Again try graphing both (that is, e^x and $c_0 + c_1x$) on your graphing calculator. What do you notice?

Again, they are clearly not the same functions, so let's try $n = 2$. Then $e^x = c_0 + c_1x + c_2x^2$. We already know that their zeroth and first derivatives are equal, so set their second derivatives equal and find $c_2 = \underline{\hspace{2cm}}$. Again graph both on your graphing calculator. What do you notice?

Repeat the procedure for $n = 3$ to find $c_3 = \underline{\hspace{2cm}}$, and describe the relationship of the two graphs.

Repeat the procedure for $n = 4$ to find $c_4 = \underline{\hspace{2cm}}$, and describe the relationship of the two graphs.

Find a general formula for $c_n = \underline{\hspace{2cm}}$. What do you expect to happen to the graphs if we take larger and larger values of n ?

Let's imagine that $\ln(x)$ can be written as a polynomial in $x - 1$, that is

$$\ln(x) = c_0 + c_1(x - 1) + c_2(x - 1)^2 + c_3(x - 1)^3 + \dots + c_n(x - 1)^n$$

for some appropriate integer $n \geq 0$. For the moment, let's pretend that $n = 0$, so $\ln(x) = c_0$. Focusing on what is happening to $\ln(x)$ at $x = 1$, we get $c_0 = \underline{\hspace{2cm}}$. Try graphing both (that is, $\ln(x)$ and the value you got for c_0) on your graphing calculator, with $xMin = -1$, $xMax = 3$, $yMin = -2$, $yMax = 2$. Describe what you find.

They are clearly not the same functions, so let's move next to $n = 1$. Then since $\ln(x) = c_0 + c_1(x - 1)$, we expect that their derivatives are also equal. Differentiate both sides and plug in $x = 1$ to find $c_1 = \underline{\hspace{2cm}}$. Again try graphing both (that is, $\ln(x)$ and $c_0 + c_1(x - 1)$) on your graphing calculator. What do you notice?

Again, they are clearly not the same functions, so let's try $n = 2$. Then $\ln(x) = c_0 + c_1(x - 1) + c_2(x - 1)^2$. We already know that their zeroth and first derivatives are equal, so set their second derivatives equal and find $c_2 = \underline{\hspace{2cm}}$. Again graph both on your graphing calculator. What do you notice?

Repeat the procedure for $n = 3$ to find $c_3 = \underline{\hspace{2cm}}$, and describe the relationship of the two graphs.

Repeat the procedure for $n = 4$ to find $c_4 = \underline{\hspace{2cm}}$, and describe the relationship of the two graphs.

Find a general formula for $c_n = \underline{\hspace{2cm}}$. What do you expect to happen to the graphs if we take larger and larger values of n ?