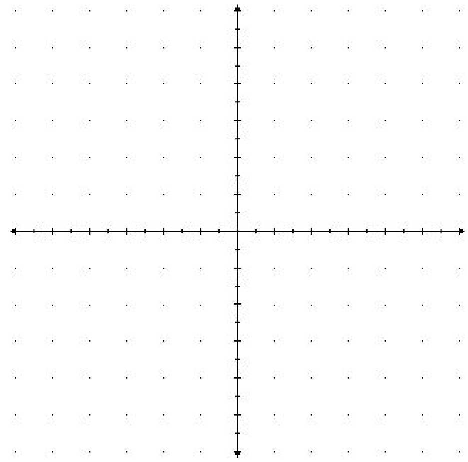
Line Integrals

1. Sketch the vector field $\vec{F} = y\vec{i} + x\vec{j}$ at right. (For æsthetics, it might help to draw $\vec{F}/5$ instead of $\vec{F}$.)



$$\textbf{Let}: \cosh t = \frac{e^t + e^{-t}}{2}, \quad \sinh t = \frac{e^t - e^{-t}}{2}.$$

2. Show that $\frac{d}{dt}[\cosh t] = \sinh t$.

3. Show that $\frac{d}{dt}[\sinh t] = \cosh t$.

4. Find a simplified expression for $\cosh^2 t + \sinh^2 t$.

5. Find a simplified expression for $\cosh^2 t - \sinh^2 t$.

6. Let C_1 be the curve $x_1(t) = \cosh t$, $y_1(t) = \sinh t$, $-1.5 \le t \le 1.5$. Sketch the curve above.

7. Based on your sketch, predict whether $\int_{C_1} \vec{F} \cdot d\vec{r_1}$ will be positive, negative, or zero. Explain.

8. $\vec{F}\left(\vec{r_1}(t)\right) =$ __________ $\vec{i} +$ __________ $\vec{j}$.

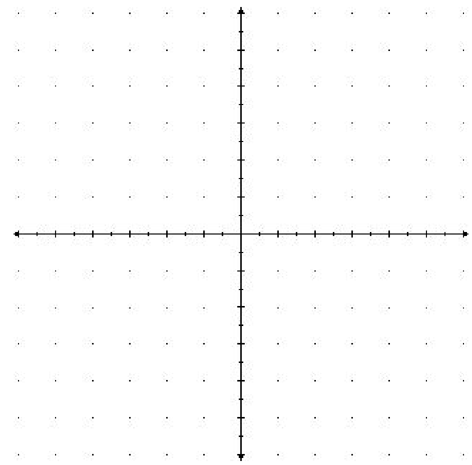
9. $\vec{r_1}'(t) = \Big($__________ $\vec{i} +$ __________ $\vec{j}\Big)dt$.

10. $\vec{F}\left(\vec{r_1}(t)\right) \cdot \vec{r_1}'(t) = ($______________________________$)dt$.

11. $\int_{C_1} \vec{F} \cdot d\vec{r_1} =$

12. Let C_2 be the curve $x_2(t) = \cos t$, $y_2(t) = \sin t$, $0 \le t \le \frac{\pi}{4}$. Repeat 6-11 for the curve C_2.

13. Sketch the vector field $\vec{G} = -y\vec{i} + x\vec{j}$ at right. (For æsthetics, it might help to draw $\vec{G}/5$ instead of $\vec{G}$.)



14. Let C_3 be the curve $\vec{r_3}(t) = \cos t\vec{i} + \sin t\vec{j}$, $0 \leq t \leq 2\pi$. Sketch C_3 at right and find $\int_{C_3} \vec{G} \cdot d\vec{r_3}$.

15. Assume that $\vec{G} = \nabla g$ for some appropriate function $g(x, y)$. Apply the FTCLI to find $\oint_{C_3} \vec{G} \cdot d\vec{r_3}$.

16. Compare your answers to 14 & 15. What conclusion can be made?

17. Let $\vec{H} = \vec{i} - 2y\vec{j}$. Pick a curve C_4 that joins the point $(1, 0)$ to $(0, 1)$. Find a parameterization for C_4.

$\vec{r_4}(t) =$

18. Calculate $\int_{C_4} \vec{H} \cdot d\vec{r_4}$.

19. Again let $\vec{H} = \vec{i} - 2y\vec{j}$. Pick a *different* curve C_5 that joins the point $(1,0)$ to $(0,1)$. Find a parameterization for C_5.

$\vec{r_5}(t) =$

20. Calculate $\int_{C_5} \vec{H} \cdot d\vec{r_5}$.

21. Compare your results on 18 & 20 with each other and with others' in the class. What conclusions and/or conjectures can be made?

22. As you probably guessed, $\vec{H} = \nabla h$ for an appropriate $h(x,y)$. Find one.

23. Use your answer in 22 and the FTCLI to calculate $\int_C \vec{H} \cdot d\vec{r}$ where C is any curve connecting $(1,0)$ to $(0,1)$.

• *We say that a vector field $\vec{F}$ is* **conservative** *or* **path independent** *if $\int_{C_i} \vec{F} \cdot d\vec{r} = \int_{C_j} \vec{F} \cdot d\vec{r}$ for all choices of C_i & C_j that connect the same endpoints.*

24. Theorem: *If $\vec{F} = \nabla f$ for some function f, then $\vec{F}$ is conservative.*

Proof: