THE TAXICAB METRIC

©2006 by Daniel J. Heath, PLU

Uses notation of <u>College Geometry</u> by David Kay

0. Imagine that we are trying to move from the point $A(0,0)$ to the point $C(2,2)$, but are only allowed to move vertically and/or horizontally (like a taxi on NS and EW roads).

a. Is there a "shortest path" from A to C? Why or why not?

b. Is there a "shortest distance" of paths from A to C? If so, what is it? If not, why not?

c. In general, find a formula for the shortest distance of paths from $A(0,0)$ to $P(x,y)$.

Definition: Denote a point in the plane by its x and y coordinates, i.e., (x,y). Then we define the *taxicab distance* from (x_1,y_1) to (x_2,y_2) to be the quantity $\mid x_2 - x_1 \mid + \mid y_2 - y_1 \mid$. Note that this is different from the "standard method" of measuring distance.

1. Prove that the taxicab distance satisfies **Axiom D-1**.

2. Prove that the taxicab distance satisfies **Axiom D-2**.

3. Prove that the taxicab distance satisfies **Axiom D-3**.

4. Prove that the taxicab distance satisfies **Axiom D-4**.

Note: Since the taxicab distance satisfies all of the metric axioms, then even if we use this "weird" method of fulfilling the **Ruler Postulate**, the plane with all of the axioms we have seen so far is a valid model of our geometry.

5. Let $A(0,0)$, $B(2,0)$, and $C(2,2)$.

a. Find AB.

b. Find $m\angle ABC$.

c. Find BC.

6. Let $X(0,-1)$, $Y(1,0)$, and $Z(0,1)$.

a. Find XY.

b. Find $m\angle XYZ$.

c. Find YZ.

7. Verify that the **SAS Hypothesis** (p. 124) has been satisfied for the correspondence $ABC \leftrightarrow XYZ$.

8. Find AC and XZ. What can we conclude about the **SAS Hypothesis**?

9. If time permits...sketch the graph of the unit taxicab circle.

10. If time permits...sketch the set of points that are equal taxicab distance from A and C