Guidelines and topics for the group project

Math 152

Spring 2010

1 Guidelines

There are many topics in the book that we will not have time to cover, which is a shame, because it would be nice to have plenty of time to look at many different applications of calculus and really see why it is such an important part of mathematics.

Since we will not have time to cover all these applications as a class, you will be required to pick one of the topics listed below, and to learn as much as possible about it, from the book, sources I will list, and others that you may find on your own. You can work alone, with a partner, or in a group of three. I do not, however, recommend working on your own, unless you have a schedule that makes it difficult to meet with others.

I will ask you to pick a topic and a partner/group, if applicable, by the beginning of March. Each group will have to pick a different one, unless there are more groups than topics. You will then write a 3-5 page paper about your topic. The paper will include a description of the topic and its relevance, mathematical formulas, explanations, graphs, and some related mathematical problems, if possible. You must type the paper, but may write the formulas and draw the graphs by hand. I will post a grading rubric for the paper later in the semester.

You will turn in the first draft of the paper on Monday, April 12. I will give you prompt feedback, and the final draft of the paper will be due on Monday, April 19, when presentations will also begin.

Each group will give a five-minute presentation in class, on a date you will sign up for beforehand. To ensure that the presentations are clear and useful to your classmates, I will ask you to do a practice run of the presentation in my office.

Think of yourself as a researcher. I am not going to give you exactly the information you need to write the paper. I will provide you with various resources, and you will have to figure out what will be relevant to you. Please ask me if you are confused, though. I will gladly help.

More detailed instructions for the different topics are given below. Once you choose a topic I will email you all the relevant links.

1.1 History of integral calculus before Newton and Leibnitz

The story of Newton's and Leibnitz's invention of calculus is rather well-known. It is less-well known that the ancient Egyptians, Greeks, and Hindus, among others, used methods of integral calculus centuries before Newton and Leibnitz. In this project, your job will be to uncover some of that history. Your paper should include the story of integration in ancient times, and the math problems that these ancient peoples knew how to solve. You have to make sure that there is enough calculus involved in the project: if you are talking about a problem that ancient Egyptians solves using Calculus, you need to solve the problem too. I would recommend talking about the discovery of Archimedes' manuscript in your in-class presentation, but you may also take a different approach. You will probably want to focus on only a small part of the history, but present it in more depth.

Some of the resources you may wish to consult:

http://www.sciencenews.org/view/generic/id/8974/title/Math_Trek_A_Prayer_for_Archimedes
http://en.wikipedia.org/wiki/Archimedes_Palimpsest
http://en.wikipedia.org/wiki/History_of_calculus#Integral_calculus (only the section on Integral Calculus and relevant links from that section)
http://www.math.tamu.edu/∼dallen/history/egypt/node4.html

1.2 The average value of a function

The average value of finitely many numbers is easy to find. But what about an infinite number? It shouldn't be a surprise that there are integrals involved int he formula. Your job for this project will be to read and understand Section 6.5, especially the graphic interpretation of the Mean Value Theorem for integrals, to solve some problems from the section (I recommend 17, 18), and then to read and interpret the following application of the average value of a function:

http://www.intmath.com/Applications-integration/9_Average-value-function.php

It is a little difficult to understand, but I will be glad to help. This is a very real, non-contrived application of calculus.

In the paper, mostly focus on Section 6.5, and give an explanation of your understanding of how the average value is used in the application. In the class presentation, give a graphic representation of the average value of integrals, and show some examples.

1.3 Hydrostatic pressure

This topic will especially be interesting to you if you are in a physics class or are majoring in physics, though hydrostatic pressure also has applications in medicine. Research information about hydrostatic force and pressure (start with a physics textbook). I have added some helpful links (one of which is not mathematical at all). Read the first part of Section 8.3 and make sure

you understand why we end up integrating to get the answer. Include the explanation in your paper. Also solve some problems from the book.

For the class presentation, talk more generally about the importance of hydrostatic pressure, and give an example.

Some solved examples are available at

http://tutorial.math.lamar.edu/Classes/CalcII/HydrostaticPressure.aspx.

Other resources:

http://www.berkeleydaily.org/issue/2009-06-04/article/33032?headline=About-the-House-Hydrostatic-Pressure-And-Why-Your-Basement-Leaks
http://www.math.uconn.edu/∼schwarz/math1122f08/HydrostaticForce.pdf
http://greenfield.fortunecity.com/rattler/46/some_basic_principles_of_fluids_.htm
http://www.enotes.com/earth-science/hydrostatic-pressure

1.4 Work

The formula for computing work needed to move an object between two points is simple when the force acting on the object is constant. Since that is often not the case, we need calculus to compute work when the force is variable. Read Section 6.4 from the book, and look at the following websites for more information:

http://www.scasd.us/teacher/jeffrey/Work, Springs, and Hooke's Law/Web Page Data/index.html
http://spiff.rit.edu/classes/phys311.old/lectures/spring/spring.html
http://www.intmath.com/Applications-integration/7_Work-variable-force.php

You will also work on a Maple worksheet, "From the Earth to the Sun." In the paper, and the in-class presentation, explain how work is calculated, why we use integrals to find work when the force is not constant, and include some examples.

1.5 Moments and centers of mass

"The center of mass is an important point on an aircraft, which significantly affects the stability of the aircraft. To ensure the aircraft is safe to fly, it is critical that the center of mass fall within specified limits. This range varies by aircraft, but as a rule of thumb it is centered about a point one quarter of the way from the wing leading edge to the wing trailing edge (the quarter chord point). If the center of mass is ahead of the forward limit, the aircraft will be less maneuverable, possibly to the point of being unable to rotate for takeoff or flare for landing. If the center of mass is behind the aft limit, the moment arm of the elevator is reduced, which makes it more difficult to recover from a stalled condition. The aircraft will be more maneuverable, but also less stable, and possibly so unstable that it is impossible to fly." (from Wikipedia)

The center of mass is a very commonly used concept in physics. The first link below shows that it is also useful in art and toy-making. Use the book, Section

8.3, and links below to understand how the center of mass is used and how it is computed. Solve some problems from the book to illustrate. Include the explanation and examples in the paper. I will also give you a Maple worksheet, "The Center of Mass of a Sculpture" to complete.

http://dev.physicslab.org/Document.aspx?doctype=3&filename=RotaryMotion_CenterMass.xml
http://ruina.tam.cornell.edu/Book/COMRuinaPratap.pdf (stop at page 86)

For the in-class presentation, explain the mathematical meaning of the center of mass and how it is related to integrals, but you also may show some of the more interesting examples from the above websites.

1.6 Centers of mass again

For this project, you will read the following paper: "Centers of the United States" which can be found at

http://users.dickinson.edu/*sim*richesod/centerus/centerus.pdf

and work through the accompanying Maple worksheet

http://users.dickinson.edu/∼richesod/centerus/

Your job in the paper will be to explain the procedures used to find the centers, as well as the area and perimeter of the U.S. For the in-class presentation, guide the class through the Maple worksheet and explain how the answers were found, stating how close these answers are to the actual ones.

1.7 Consumer surplus

This is a common concept in economics, and there are many online resources explaining it. If you are taking an economics class, or have taken one in the past, you have probably encountered this concept, and should use an economics textbook for the topic. It is explained in Section 8.4, and other resources are below:

http://en.wikipedia.org/wiki/Economic_surplus#Consumer_surplus
http://tutor2u.net/economics/revision-notes/as-markets-consumer-surplus.html

I will give you a few pages from a different textbook that gives a better explanation of consumer surplus. Also do a search on Google images for "Consumer surplus." You will get a lot of interesting results.

For the paper, make sure you understand and are able to explain why consumer surplus is computed with an integral, and solve some sample problems. For the class presentation, talk about whatever you think will be most interesting to your classmates.

1.8 Blood flow and cardiac output

This topic will especially be relevant to you if you intend to go into medicine. Section 8.4 briefly talks about blood flow and cardiac output. Make sure you read and understand this section. However, for this topic, your main goal will be to read and understand this excellent paper:

http://www.brynmawr.edu/math/people/rhughes/6.pdf,

called "Math in the ICU." Other sites you may want to look at:

http://en.wikipedia.org/wiki/Cardiac_output
http://illuminations.nctm.org/LessonDetail.aspx?ID=U136.
medskip

For your paper, summarize the mathematics in the paper you read, and include some solved examples. In the presentation, share with the class what cardiac output is, how it is found, and briefly talk about the mathematics used in medicine, as described in the paper.

1.9 Probability

Unlike probability we learn in middle and high school, the "real world" probability theory is based on integrals. Read Section 8.5 from the book, and a section from another textbook I will provide you with, and present it in your paper. Include a discussion about averages and the normal distribution. Solve some sample problems. You will also work on a Maple worksheet, "SAT Scores." For the in-class presentation, explain probability distributions through an example, and briefly discuss normal distribution.

The following link may also be useful:

http://www.math.montana.edu/frankw/ccp/multiworld/multipleIVP/probability/learn.htm

1.10 Direction fields and Euler's method

We will only spend a short amount of time on direction fields in class, and will not do Euler's (read as "Oyler," not "Uler") method at all. For the paper, I will give you some worksheets with direction fields to work on: "Direction fields," "Using direction fields," and "Drawing solution curves"; you will also read from the book about Euler's method, and explain why it works. Finally, I will give you Maple worksheet, "Slope Fields" and "Evaluating Function Values with Euler's Method." In class, you will present to your classmates an example using Euler's method. Useful links:

http://en.wikipedia.org/wiki/Euler_method
http://www.csun.edu/∼hcmth018/EuM.html

1.11 Models of population growth

In the 18th century, Thomas Malthus, an English scholar, made a prediction that the Earth population would grow exponentially, while food supply would grow linearly, and that, therefore, eventually, humans would be doomed. There is disagreement on the nature of growth of the human population. Read Section 9.4 to look at different models for population growth, and consult the links I have posted below:

http://en.wikipedia.org/wiki/Population_growth
http://www.math.duke.edu/education/postcalc/growth/growth2.html
http://home.comcast.net/ sharov/PopEcol/lec5/explog.html

and then write a summary of the exponential and logistic model, as well as another one, of your choosing; provide formulas and graphs for each; and give examples (real-life data would be especially interesting – some of the links I have provided have some real-life data). I will also give you a Maple worksheet, "Comparing Population Models." You will also work on some related problems that I will provide you: "Spread of a rumor" and "Population." In class, give a brief presentation of the exponential and logistic model, and show graphs and formulas, and an example.

Note: This project can potentially be broken into two if more than one group is interested in working on it.

1.12 Predator-prey models

Predator-prey models are one of the best known applications of calculus: we can use differential equations to predict populations of animal species that are in a predator-prey relationship. You can read about this in Section 9.6, and you can also consult the following websites:

http://home.messiah.edu/ deroos/CSC171/PredPrey/PPIntro.htm
http://www.globalchange.umich.edu/globalchange1/current/lectures/predation/predation.html

However, the main part of this assignment will be using Maple to understand the simplest, Lotka-Volterra model. Just follow the steps of the following module:

http://www.math.duke.edu/education/ccp/materials/diffeq/predprey/contents.html

For your paper, just answer all the questions in the module. For the in-class presentation, briefly explain the model, and show the class some graphs, both of actual populations and of simulations.