Some initial proofs

You may want to use Theorem 1.7 to solve these problems.

1. Suppose that a is a real number. Prove that if $a^3 > a$ then $a^5 > a$.

2. Consider the following theorem:

   Suppose x is a real number and $x \neq 4$. If $\dfrac{2x-5}{x-4} = 3$, then $x = 7$.

   (a) What's wrong with the following proof of the theorem?
   Suppose $x = 7$. Then $\dfrac{2x-5}{x-4} = \dfrac{2(7)-5}{7-4} = 9/3 = 3$. Therefore if $\dfrac{2x-5}{x-4} = 3$, then $x = 7$.

   (b) Give a correct proof of the theorem.

3. Prove that if $x^2 + y = 13$ and $y \neq 4$, then $x \neq 3$.

4. Suppose that x and y are real numbers. Prove that if $x^2 y = 2x + y$, then if $y \neq 0$, then $x \neq 0$.

5. Suppose A, B, and C are sets, and $A \setminus B \subseteq C$. Then $A \setminus C \subseteq B$.

6. Prove that for every real number x, if $x > 0$ then there is a real number y such that $y(y+1) = x$.

7. Suppose x is an integer. Prove that x is even iff x^2 is even.

8. Show that $\sqrt{2}$ is not a rational number.