

**Math 245, February 24, 2010**

**Quiz 1**

You may work in groups and you may use the textbook and notes. Pick two out of the four problems. Finish the other two for homework.

1. Verify, using both truth tables, and Theorem 1.7, that  $(p \rightarrow q) \vee (q \rightarrow p)$  is a tautology. Hint: you may need to use the tautology  $p \vee \neg p$ .
2. Show that it is not the case that if  $\exists xP(x) \rightarrow \exists xQ(x)$  then  $\exists x(P(x) \rightarrow Q(x))$ . Hint: find convenient interpretations for  $P$  and  $Q$ , and an  $x$  for which the first statement is true, but the second is false.
3. Write the statement *The square of every even number is divisible by 4* as a first-order sentence (in other words, you can use quantifiers only on numbers; assume that the universe is all natural numbers.) Prove this statement. Which rule are you using?
4. Show that the power set of a set with  $n$  elements has  $2^n$  elements.