Name: ____

Math 245, Discrete Structures
March 12, 2010
Test 1

This table it used for grading only. Do not write anything in it.

Problem	Points	Out of
1.		20
2.		15
3.		10
4.		15
5.		16
6.		24
Total		100

The first five problems are to be done individually. You will have ten minutes to discuss the last problem with your group, but you will have to write it up on your own.

1. (5 x 4 points) In fuzzy logic, a proposition has a truth value that is a number between 0 and 1 inclusive. A proposition with a truth value of 0 is false and one with a truth value of 1 is true. Truth values between 0 and 1 indicate varying degrees of truth. For example, the truth value 0.8 can be assigned to the statement "Ginger is happy," since Ginger is happy most of the time, and the truth value 0.4 can be assigned to the statement "Fred is happy," since Fred is happy slightly less than half the time.

   (a) What should be the truth value of the negation of a proposition? What is the truth value of the proposition "Ginger is not happy?"

   (b) What should be the truth value of the conjunction of two propositions? What is the truth value of the proposition "Ginger is happy and Fred is happy?"

   (c) What should be the truth value of the disjunction of the two propositions? What is the truth value of the proposition "Ginger is happy or Fred is happy?"

   (d) What is the truth value of the statement "Neither Ginger nor Fred is happy?"

   (e) What is the truth value of the proposition "If Fred is happy then Ginger is happy?" (Hint: Use an equivalent statement for "if... then.")

2. (3 x 5 points) Fuzzy sets are also used in artificial intelligence. Each element of the universal set U has a degree of membership, which is a real number between 0 and 1 (including 0 and 1) in a fuzzy set S. The fuzzy set S is denoted by listing the elements with their degrees of membership (elements with 0 degree of membership are not listed). For instance, we write {0.6 Adams, 0.9 Rodriguez, 0.4 Yu, 0.1 Lovejoy, 0.5 Nasir} for the set F (of famous people) to indicate that Adams has 0.6 degree of membership etc. (so that Rodriguez is the most famous and Lovejoy is the least famous of these people). Also suppose that R is the set of rich people, with R={0.4 Adams, 0.8 Rodriguez, 0.2 Yu, 0.9 Lovejoy, 0.7 Nasir}.

   (a) How would you define the complement S^C of a fuzzy set S? What would be the degree of membership of each of the elements of S^C? Find F^C (the fuzzy set of people who are not famous) and R^C (the fuzzy set of people who are not rich).

   (b) The union of two fuzzy sets S and T is the fuzzy set $S \cup T$. How would you define the degree of membership of an element in $S \cup T$? Find the fuzzy set $F \cup R$ of rich or famous people.

   (c) The intersection of two fuzzy sets S and T is the fuzzy set $S \cap T$. How would you define the degree of membership of an element in $S \cap T$? Find the fuzzy set $F \cap R$ of rich and famous people.

3. (10 points) Find a proposition in terms of p, q and r that is true when exactly two of the three variables are true, and is false otherwise. (Hint: It may help to use a truth table and a past homework problem.)

4. (3 x 5 points) Recall that "nor" ($\downarrow$) is "not or," that is, $p \downarrow q \equiv \neg(p \vee q)$.

   (a) Show that $p \downarrow p$ is logically equivalent to $\neg p$.

   (b) Show that $p \downarrow q$ and $q \downarrow p$ are logically equivalent.

   (c) Is $\downarrow$ associative? That is, is $p \downarrow (q \downarrow r) \equiv (p \downarrow q) \downarrow r$?

5. (6+10 points) The notation $\exists!\, x\, P(x)$ denotes the proposition "There exists a unique x such that $P(x)$ is true."

   (a) If x ranges over the set of integers, what are the truth values of

      i. $\exists!\, x\, (x > 1)$

      ii. $\exists! x\, (x^2 = 1)$

      iii. $\exists! x\, (x - 3 = 2x)$

      Briefly explain each answer.

   (b) What are the truth values of the following statements?

      i. $\exists!\, x\, P(x) \rightarrow \exists x P(x)$

      ii. $\forall x\, P(x) \rightarrow \exists!\, x\, P(x)$

      iii. $\exists! x\, \neg P(x) \rightarrow \neg \forall x P(x)$

      Briefly explain each answer.

   (c) (EXTRA CREDIT, 5 points) Express the quantification $\exists!\, x\, P(x)$ using universal quantification, existential quantification, and logical operators.

6. (4 x 6 points) Prove or disprove the following claims.

   (a) All primes are odd.

   (b) The sum of a rational and irrational number is irrational.

   (c) The product of two irrational numbers is irrational.

   (d) $n^4 - 1$ is divisible by 5 when n is not divisible by 5.

   (e) (EXTRA CREDIT, 6 points) For every positive integer n, there is an integer divisible by more than n primes.