

Math 245, Test 1 supplement

1. Show that the sum of a rational and irrational number is irrational.
2. Show that if n is not divisible by 5, then $n^4 - 1$ is divisible by 5.
3. Show that the sum of five consecutive numbers is divisible by 5.
4. Show that each prime number other than 2 and 3 can be written as either $6k + 1$ or $6k + 5$. Use this to show that the square of a prime number (other than 2 or 3) has remainder 1 when divided by 6.
5. Show that the product of any four consecutive numbers is divisible by 24.
6. Show that a number n is prime if and only if the sum of its factors is $n + 1$.
7. Show that the difference between any two primes larger than 2 cannot be an odd number.
8. For any integer n , $n^2 - 2$ is not divisible by 4.
9. For all prime numbers a , b , and c , $a^2 + b^2 \neq c^2$.
10. Show that for every positive integer n , there is a number divisible by more than n primes.
11. Let x and y be integers. If x and y satisfy the equation $3x + 5y = 153$, show that at least one of x and y has to be odd.
12. Show that for every positive real number x , $x + 1/x \geq 2$.
13. Show that if $n \bmod 4 = 2$, then n is not a perfect square.
14. Show that there is a rational number that is strictly between the square root of 10^{100} and square root of $10^{100} + 1$.
15. Show that for any two numbers a and b , $\|a + b\| \leq \|a\| + \|b\|$. Note that a and b can be both positive or negative.