Math 245, March 1, Sections 2.3 and 2.4

More practice with direct proofs

Problems to solve: 26, 41, 44 from 2.3.

Indirect proofs

(modified from http://zimmer.csufresno.edu/∼larryc/proofs/proofs.html)

Proof by Contradiction In a proof by contradiction we assume, along with the hypotheses, the logical negation of the result we wish to prove, and then reach some kind of contradiction. That is, if we want to prove "If P, Then Q", we assume P and Not Q. The contradiction we arrive at could be some conclusion contradicting one of our assumptions, or something obviously untrue like $1 = 0$.

Proof by contradiction is often used when you wish to prove the impossibility of something. You assume it is possible, and then reach a contradiction.

Proof by Contradiction is often the most natural way to prove the converse of an already proved theorem.

Proof by Contrapositive Proof by contrapositive takes advantage of the logical equivalence between "P implies Q" and "Not Q implies Not P". For example, the assertion "If it is my car, then it is red" is equivalent to "If that car is not red, then it is not mine". So, to prove "If P, Then Q" by the method of contrapositive means to prove "If Not Q, Then Not P".

How Is This Different From Proof by Contradiction? The difference between the Contrapositive method and the Contradiction method is subtle. Let's examine how the two methods work when trying to prove "If P, Then Q".

- Method of Contradiction: Assume P and Not Q and prove some sort of contradiction.
- Method of Contrapositive: Assume Not Q and prove Not P.

The method of Contrapositive has the advantage that your goal is clear: Prove Not P. In the method of Contradiction, your goal is to prove a contradiction, but it is not always clear what the contradiction is going to be at the start.

Look at Examples 2.25 and 2.29.

Problems to do: 7, 15, 28, 31, 33 from 2.4.