

Combinatorics problems

Sections 6.1-6.3

Math 245, Spring 2010

1 Initial activity: PLAYING CARDS

In the following, hands of cards are dealt from a well shuffled pack of 52 cards.

1. How many different poker hands of 5 cards:
 - (a) consist of all hearts
 - (b) consist of cards of the same suit (flush)
 - (c) contain 4 of a kind eg. 4 kings and another card
 - (d) contain 3 of one kind and 2 of another kind (full house) eg. 3 kings and 2 fives ?
2. Find the probability of getting each of the types of hands in the problem above. (There will be more on probability on the next worksheet.)
3. Check this out: <http://pokerroomreview.com/poker-news/poker-a-game-of-skill/>

2 How to solve it

There are four main "strategies" for solving the problems:

- Multiplication principle: A mans wardrobe consists of 5 sport coats, 3 dress slacks, and 2 pairs of shoes. Assuming they all match, in how many ways can he select an outfit?
- Addition principle: How many bytes have 2 or 3 zeros?
- Permutations:A man has a nickel, dime, quarter, and a half dollar. How many ways can the waitress be tipped if he gives her two coins? At least two coins?

- Combinations: In a group of 10 people, each person shakes hands with everyone else once. How many handshakes are there?

A few tutorials:

- <http://www.ucc.edu/NR/rdonlyres/D55520DE-CD3A-42CB-8CEE-471329AA6A91/0/CountingM>
- <http://www.mathsisfun.com/combinatorics/combinations-permutations.html>
- <http://www.brainjammer.com/math/combinatorics/>

3 Problems

Now try these. In each case specify what you are using to solve the problem.

1. Ten cars are in a race. How many ways can we have first, second, and third place?
2. Ten cars are in a race. Three of the ten will qualify for the next race. In how many different ways can this happen?
3. How many ways can a True-False test be answered if there are 6 questions?
4. License plates consist of two letters and five digits. How many different license plates can be made?
5. A city council is composed of 5 liberals and 4 conservatives. A delegation of three is to be selected to attend a convention. How many delegations are possible? How many could have all liberals? How many could have 2 liberals and one conservative?
6. The school board consists of seven members, if the first person selected is the president, the second is the vice president and the third is the treasurer, how many ways can the officers of the board be chosen?
7. Two witnesses to a bank robbery had different memories of the license plate on the getaway car. Both agreed that the plate consisted of 6 digits. However, one noticed that there were exactly 2 ones, and the other noticed that there were exactly 3 nines on the plate. To be safe, and realizing that one of the witnesses might be mistaken, the police want to consider both possibilities. How many license plates consisting of 6 digits (0 to 9) have exactly 2 ones or 3 nines?

8. The letters ENGLISH are arranged to make 7 letter words without repetition. How many words can be made if: a) there are no restrictions b) the first letter is E c) E is the first or last letter d) the first and last letters are vowels e) the first letter is a consonant f) the first and last letters are consonants g) the E and I occur next to each other in the order EI h) the E and I occur next to each other i) the E, N and G occur next to each other?
9. Three married couples (the Smiths, Jones and Browns) sit on a bench. In how many ways can this be done if: a) there are no restrictions b) the Smiths sit at the ends c) the Smiths sit together d) each married couple sits together?
10. Three married couples (the Smiths, Jones and Browns) sit at a circular table. In how many ways can this be done if: a) there are no restrictions b) the Smiths sit together c) each married couple sits together
11. a) How many 4 digit integers are odd? b) How many 4 digit integers are multiples of 5? 11. How many 4 digit integers can be made from the digits 1, 3, 4, 5, 6, 7 if: a) repetition is not allowed b) repetition is allowed?
12. How many even 4 digit integers can be made from the digits 1, 3, 4, 5, 6, 7 if: a) repetition is not allowed b) repetition is allowed?

4 Sample problems

You do not have to solve these problems. They are examples of different types of problems:

MULTIPLICATION PRINCIPLE

- How many even 3-digit positive integers can be written using the digits 1, 3, 4, 5, and 6?
- In how many ways can you select 4 cards, one after the other, from a 52-card deck: a) if the cards are returned to the deck after being selected? b) if the cards are not returned to the deck after being selected?
- How many 7-digit phone numbers can be created if the first digit cannot be 0 or 1, the second must be a 5, and the third must be a 3 or 4?

- How many positive odd integers less than 10,000 can be written using the digits 3, 4, 6, 8, and 0?

PERMUTATIONS

- In how many ways can 11 books be arranged on a shelf: a) using all the books? b) using 4 of the books?
- In how many ways can the letters of the word MONDAY be arranged using a) all six letters? b) using 3 letters at a time?

COMBINATIONS

- A sample of 4 Ipods taken from a batch of 100 Ipods is to be inspected. How many different samples can be selected?
- In how many ways can a 5-card hand be dealt from a standard deck of cards?
- In how many ways can a committee of 7 be chosen from 9 girls and 8 boys if a) all are equally eligible? b) the committee must include 4 girls and 3 boys?