

Fibonacci Induction Problems

Prove:

1. For every $n \geq 1$, f_{3n} is even.
2. For every $n \geq 1$, f_{4n} is divisible by 3.
3. For every $n \geq 1$, $f_n^2 + f_{n+1}^2 = f_{2n+1}$.
4. For $n \geq 2$, $f_{n-1} \cdot f_{n+1} = f_n^2 + (-1)^n$.

Assume $f_1 = 1$.