Math 245: Practice Problems for the second test (and beyond)

1. The set Σ^* of strings over the alphabet Σ can be defined recursively by $\lambda \in \Sigma^*$ where λ is the empty string containing no symbols, and $wx \in \Sigma^*$ whenever $w \in \Sigma^*$ and $x \in \Sigma^*$. Give a recursive definition of $l(w)$, the length of the string w.

2. The reversal of a string is the string consisting of the symbols of the string in reverse order. The reversal of a string is denoted by w^R. Give a recursive definition of a reversal of a string (Hint: first define the reversal of the empty string. Then write a string w of length $n+1$ as xy where x is a string of length n and express the reverse of w in terms of x^R and y).

3. A single pair of rabbits (male and female) is born at the beginning of a year. Assume the following conditions:

   (a) Rabbit pairs are not fertile during their first two months of life, but thereafter give birth to three new male/female pairs at the end of every month.

   (b) No rabbits die.

   Let s_n be the number of pairs of rabbits alive at the end of month n, for each integer $n \geq 1$, and let $s_0 = 1$. Find a recurrence relation for $s_0, s_1, \ldots$. How many rabbits will there be at the end of the year?

4. Use induction to show that a set with n elements has 2^n subsets.

5. Show that for $n \geq 4$, $2^n \leq n!$.

6. Show that the relation R, consisting of all pairs (x, y) where x and y are bit strings of length three or more that agree except perhaps in their first three bits, is an equivalence relation on the set of all bit strings.

7. Let S be the set of fractions: $S = \{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\}$ Define a relation R on S by: $\frac{a}{b} R \frac{c}{d}$ iff $ad = bc$.

8. Assume that there are n people in the room. Ignoring leap years, what is the mathematical formula for the probability that no one else in the room shares your birthday?

9. In the North Carolina Lottery Cash-5 Game, the same five numbers [4, 21, 23, 24, 39] were randomly selected on July 9, 2007, and July 11, 2007. Given that there are 39 numbers that can be selected without replacement, what is probability of at least one matching set of 5 numbers in a given set of 3 drawings?

10. The Medical Diagnosis Problem: Assume that a test to detect a disease whose prevalence is (1/1000) has a false positive rate of 5and a true positive rate of 100%. What is the probability that a person found to have a positive result actually has the disease assuming that you know nothing about the persons symptoms?