The Fundamental Theorem of Calculus (FTC)

Theorem 1 ***(FTC part 1)*** *Let $f(x)$ be a continuous function on $[a,b]$ for some real numbers a and b with $a < b$. Then $\int_a^x f(t)\ dt$ is a continuous function of x on $[a,b]$, and a differentiable function of x on (a,b), with*

$$\frac{d}{dx}\int_a^x f(t)\ dt = f(x).$$

Theorem 2 ***(FTC part 2)*** *Let $F'(x) = f(x)$ on $[a,b]$ for some real numbers a and b with $a < b$. Then*

$$\int_a^b f(x)\ dx = F(b) - F(a).$$

That is, if $F(x)$ is ***any*** *antiderivative of $f(x)$, then the integral of $f(x)$ from a to b is just the difference in the values of the antiderivative F evaluated at the endpoints a and b. Stated another way, if $F(x)$ is any solution to*

$$\int f(x)\ dx,$$

then

$$\int_a^b f(x)\ dx = F(b) - F(a).$$

Theorem 3 ***(Chain Rule for Derivatives of Integrals)*** *Let $f(x)$ be a piecewise continuous function on some interval containing the values of the differentiable functions $a(x)$ and $b(x)$. Then*

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)\ dt = f\left(b(x)\right)\cdot\frac{db}{dx} - f\left(a(x)\right)\cdot\frac{da}{dx}$$

Example:

$$\frac{d}{dx}\int_{3x}^{x^2} \cos(t)\ dt = \cos\left(x^2\right)\cdot\frac{d(x^2)}{dx} - \cos\left(3x\right)\cdot\frac{d(3x)}{dx} = 2x\cos(x^2) - 3\cos(3x)$$