Capstone Abstracts for Spring 2023

Justin Chase

Making Mathematical Roses

Plotting certain trigonometric curves based on the polar coordinate system creates interesting patterns. When expressed in the form $r = cos(c * \theta)$ where c is an integer, we call these curves Rhodonea curves or Rhodonea roses. The parameter c can also be chosen to be rational, $c = a/b$, making the new formula $r = cos((a/b) * \theta)$. We can analyze these new roses to find how changing any of the parameters affects the shapes and sizes or roses produced by the curves. We can use the information gained to make specific shapes to emulate real flowers or other designs. Finally, we can link the formula for these curves to a more general formula for polar curves.

Alexandra De Fermin Cortes

Is there an Association Between a Cystic Fibrosis Patients' Demographic Variables and their Spirometry Measurements? An Application of Logistic Regression

Abstract: Cystic Fibrosis (CF) is an inherited lung disease caused by a mutation in a single gene encoding the cystic fibrosis transmembrane conductance regulator protein. Historically, lung functions have been assessed in clinical settings with the help of a spirometer to measure the volume, capacity, and flow of air inhaled and exhaled. Due to SARS-CoV-2, the use of telehealth has become an important form of patient treatment, including conducting spirometry tests at home. The accuracy and similarity of at-home spirometry to clinical spirometry is unknown. In particular, it is unknown whether particular races or genders tend to have more accurate spirometer measurements when using the at-home spirometry testing. If such differences exist, targeted trainings could be conducted to help improve the viability of at-home spirometry. Thus, this study is conducted to see if there is an association between a patient's demographic variables and their spirometer measurements (clinical versus home spirometry). We will use race and gender as categorical response variables to illustrate logistic regression.

David Gilman

The World of Polyominoes: Topics of Interest and Methods

Polyominoes have captured the imagination of both academic and amateur mathematicians for decades. They have also frequently appearing in Martin Gardner's famous Mathematical Games. In this capstone presentation, I will explore the mathematics behind this intriguing recreational construction, and explore problems that may be encountered along the way. My overview of the subject will include investigations in tiling, various techniques of proof, and simple puzzle formation, as well as some interesting examples.

Matthew Helmer

Sturm-Liouville Theory and Application

We learn in elementary calculus that it is possible to express a "well-behaved" function *f(x)* as an infinite series known as the Taylor series, a sum of powers of *x* along with derivatives of *f* evaluated at a point. In similar fashion, it is possible to express a well-behaved *f(x)* as an infinite series of other types of functions, and these expressions have different applications depending on the sets of functions utilized. For instance, we can use the Fourier series to build a function out of sine waves, and this was originally utilized in order to solve the heat equation, or the differential equation modeling heat flow.

The sets of functions that you can do this with are special, and while all these sets of functions may appear different from one another, there is a general framework called Sturm-Liouville theory that unites all of them together and describes the conditions placed upon each. We will discuss this framework, and along the way, we will discuss what it means for a set of functions to be orthogonal and complete, we will illustrate some examples of these sets and their properties, and we will introduce Hilbert spaces, the setting in which much of quantum mechanics takes place.

Kioni Kamau

Exploring Causal Effects and Inference: An Analysis of Average Treatment Effects

Causal inference is an important field in statistics and data science, with significant applications in various areas, such as healthcare, social sciences, and economics. This capstone project focuses on the analysis of causal effects and inference, with a particular emphasis on the average treatment effect, structural causal models, and conditional exchangeability. The project begins with an overview of the concepts of causality, counter-factuals, and causal inference. Then, it delves into the average treatment effect, which measures the difference in outcomes between two groups, one of which receives a treatment and the other of which does not. Structural causal models are introduced as a framework for representing causal relationships between variables, allowing for causal inference to be made through graphical models. Finally, conditional exchangeability is explored as a key assumption for estimating causal effects. The project also presents examples of the applications of these concepts in real-world scenarios. Overall, this capstone project provides a comprehensive overview of causal effects and inference, demonstrating the importance of these concepts for understanding and making informed decisions based on data.

Zhanna Kukosh

Preparing Students to Succeed in Geometry

Geometry is an extremely important subject that students are required to take in high school. Unfortunately, it is disliked by many and has a bad reputation. Many times, students simply are not ready to take the course because they don't have enough prior experience with geometry. The Dutch mathematics educators Dina van Hiele-Geldof and Pierre van Hiele came up with a model that explains why so many struggle with geometry. Their model includes 5 levels. The Dutch educators discovered that geometry is taught at level 4, but most students arrive at level 1. This is an extremely large gap that geometry teachers struggle to fix. In particular, this

means that students struggle with proofs, which is the focus of level 4. In this presentation we will discuss different types of proofs students learn, what the different levels are and how teachers can help students advance through the levels.

Jackie Lindstrom

Principal Components Analysis: Using Mathematics to Extract Underlying Structures in the Iraqi Migration Survey Data

Principal Components Analysis (PCA) is a statistical technique used across a variety of scientific disciplines to gain better insight into the underlying structure of a data set. Since our minds have a difficult time comprehending a large number of variables, we often seek to reduce the dimension of complicated data sets. PCA is a mathematical tool used to reduce the dimension of a data set in the hope of obtaining useful and understandable takeaways. PCA can be helpful in identifying patterns and performing dimension reduction. PCA uses the vectors in the data table, each vector representing a different variable, and finds a set of new, orthogonal vectors. The original vectors can be expressed as linear combinations of the new principal components. The primary principal component captures the largest variance, or spread, and therefore “explains” the largest amount of information from the original data table. The second principal component will be orthogonal to the first, and usually contains less information.

PCA will be performed using R on a data set. This data set holds the results of a survey sourced from the International Organization for Migration. The survey was conducted in Iraq asking internally displaced persons about their access to certain needs, such as distance from clinics, access to clean water, etc. Before performing PCA, this data set needs to be prepared by addressing missing data points, normalizing the vectors and centering the vectors. Performing PCA on this dataset could help the International Organization for Migration detect patterns and visualize underlying structures in the needs of displaced persons. Additionally, PCA could be used to help improve the survey design, as results can suggest where to reduce redundancy in questions or where to add additional questions to better capture the experiences and needs of this population.

Logan Margo

Data Analysis of Minority Suspension Disparities in School Districts

This research paper explores the issue of disproportionate suspensions of minorities in school districts through a broad scope of data analysis techniques. The study employs Principal Component Analysis (PCA) to reduce the dimensionality of the data and identify the key variables that contribute to suspension disparities. Additionally, multiple regression models are utilized to predict suspension rates in various districts based on specific factors such as race, socio-economic status, and school policies. The results of this analysis demonstrate that minority students are disproportionately affected by suspensions, and that certain policy changes and interventions could lead to a reduction in suspension rates. This study emphasizes the importance of utilizing data analysis techniques to identify and address disparities in educational outcomes, and provides insights into potential solutions to promote equity in schools. This research also aims to conduct analysis on a state and national level to provide a more comprehensive understanding of the issue. By examining suspension data from multiple states and comparing the findings with national trends, this study will provide insights into the varying degrees of disparity across different regions.

Spencer McCray

Existence of Compositional Roots of One-to-one Functions

A compositional nth root of a function g is the function f where f composed with itself n times is the function g, written symbolically as $f^n = g$. For the sake of simplicity, we shall primarily look at one-to-one functions. The existence of a compositional root is dependent upon the orbits of the function. Through examples we will explore when functions have compositional roots, when and why those roots are or are not continuous, as well as the construct of a non-trivial compositional root.

Holden C. Smith

Constrained optimization: The Lagrange Multiplier Method and The Envelope Theorem

Constrained optimization is a key tool used throughout business and economics. Being able to quickly find a local or global extremum of a function in a model is valuable, and one of the most common ways to do this is with the Lagrange multiplier Method (LMM).

In this paper, I will describe, in detail, the Lagange multiplier method and some of its uses in modern economics. Solving *constrained* optimization problems is important in economics since there are many models where optimization is needed but there is some constraint on the model such as restrictions on utility or on the total money available. With our understanding of the LMM, we will look at a special case called the Envelope theorem.

Ryan Stracke

Basic Blackjack Strategy and Card Counting

The game of blackjack is a player versus dealer card game where the player attempts to have a higher hand than the dealer without exceeding 21. While playing blackjack, players can maximize their profits by following a basic strategy: counting cards and adjusting one's betting based on the card count. The goal of this paper is to explain the reasoning behind basic strategy and card counting by using mathematical probability. A second goal is to estimate approximately how likely a player is to profit from playing blackjack with either a perfect basic strategy or an adjusted strategy.

In order to determine the likelihood of a player winning, it will be useful to know the likelihood of every combination of player hand and dealer's face-up-card. Using mathematical probability, we can determine what is a "favorable hand" and what is a "favorable up-card" in addition to what is a conjunction of the two. On top of this, we can adjust what is considered favorable based upon player information from counting the cards. Upon knowing what is considered favorable, we can find a general idea of a player's likelihood of getting a favorable hand.

Wyatt Tayon

Multiple Logistic Regression and Student Retention at Pacific Lutheran University

For a university to continue to successfully operate, it must be able to retain its student population through graduation. Retaining students beyond their freshman year is necessary since higher retention rates can be attractive to new and potential students. This paper examines Pacific Lutheran University's student retention rates to see what factors correlate to higher retention. The areas of focus when analyzing retention are the student's gender and/or race. As a control, we will look at other possible variables such as participation in a varsity sport, whether the student is the first in their family to attend college, whether the student is employed on campus, whether the student lives on campus, and how much the student had their tuition discounted from scholarships. The data utilized was provided by the Office of the Provost at Pacific Lutheran University. The student data ranges from 2012 to 2021. To analyze the data, the statistical program R will be used by creating a multiple logistic regression model to find correlations between the variables mentioned and student retention.

Michael Wilcken

Solving *n x n x n* Rubik's Cubes with Only Approximately One Algorithm

The Rubik's Cube is thought of as a deceptively simple puzzle with quintillions of possible states that only "smart" people can solve. Competitions are common to see how quickly and efficiently a scrambled cube can be returned to the solved state. Competitors memorize multiple algorithms that rearrange slices of the cube at one time.

I have created a singular, short, general algorithm that can be used to solve a cube of any size. This algorithm does not prioritize speed of solving, but it does minimize the amount of memorization needed. It also allows for the solution of any subgroup of the Cube without modification of the position or orientation of pieces (cubies) in the rest of the cube. This makes cubes with printed pictures or unusually shaped pieces easier to solve.